立方体の回転







First, we consider the axis of the rotation.

f: orthogonal transformation



f: orthogonal transformation 直交変換



f: orthogonal transformation 直交変換

Assume that f maps P(1, 1, 1)to point P' on the x-axis.



f: orthogonal transformation 直交変換





















From
$$\vec{u}_1 \cdot \vec{u}_2 = 0$$

 $\vec{u}_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1\\1\\1 \end{pmatrix}, \ \vec{u}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1\\1\\0 \end{pmatrix}, \ \vec{u}_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1\\1\\0 \end{pmatrix}, \ \vec{u}_4 = \frac{1$

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→問1

Put
$$T = (\vec{u}_1 \ \vec{u}_2 \ \vec{u}_3)$$

= $\begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{pmatrix}$.





for the fundamental vectors $\vec{i}, \ \vec{j}, \ \vec{k}$.

基本ベクトル... 座標軸に平行(正の向き)で大きさ1のベクトル



→問 2

Since
$${}^tTec{u}_1 = ec{i}, \; {}^tTec{u}_2 = ec{j}, \; {}^tTec{u}_3 = ec{k},$$

we can use the orthogonal matrix



 \boldsymbol{z}

which represents f.

$$T = \begin{pmatrix} \vec{u}_1 & \vec{u}_2 & \vec{u}_3 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{pmatrix}$$

Each vertex transformed by f, the cube is rotated around the x-axis.



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f maps segment QR to Q'R'.



f maps segment QR to Q'R'.















→問 3

















Equation of Hyperbora \boldsymbol{z} Consider intersection of face of this cube and plane x = c, -*Y* \boldsymbol{x} 8/19 \triangleleft \triangleright \triangleright
Equation of Hyperbora \boldsymbol{z} Consider intersection of face of this cube and plane x = c, y \boldsymbol{x} 9/19 \triangleright \triangleleft \triangleright

Equation of Hyperbora \boldsymbol{z} Consider intersection of face of this cube and plane x = c, -y \boldsymbol{x} 10/19 \triangleleft \triangleright \triangleright

Equation of Hyperbora \boldsymbol{z} Consider intersection of face of this cube and plane x = c, -Y \boldsymbol{x} 11/19 \triangleleft \triangleright \triangleright

Equation of Hyperbora \boldsymbol{z} Consider intersection of face of this cube and plane x = c, -y \boldsymbol{x} 12/19 \triangleleft \triangleright \triangleright



Equation of Hyperbora \boldsymbol{z} Consider intersection of face of this cube and plane x = c, -y \boldsymbol{x} 14/19 \triangleright \triangleleft \triangleright











Consider intersection of face of this cube and plane x = c, ...with circles while rotating it.



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6/19

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Consider intersection of face of this cube and plane x = c...with circles while rotating it.

The radius of circle is distance between the point on the segment Q'R' and x-axis.



Consider intersection of face of this cube and plane x = c...with circles while rotating it.

The radius of circle is distance between the point on the segment Q'R' and x-axis.

半径は線分 $\mathbf{Q'R'}$ 上の点 (X, Y, Z)と x軸上の点 (X, 0, 0)との距離 $\sqrt{Y^2 + Z^2}$



Consider intersection of face of this cube and plane x = c...with circles while rotating it. The radius of circle is

distance between the point on the segment Q'R' and x-axis.

半径は線分 $\mathbf{Q'R'}$ 上の点 (X, Y, Z)と x軸上の点 (X, 0, 0)との距離 $\sqrt{Y^2 + Z^2}$ →問 4
















































Equation of Hyperbora

$$z^{2} = \left\{-\frac{1}{\sqrt{2}}(t+1)\right\}^{2} + \left\{-\frac{1}{\sqrt{6}}(t-3)\right\}^{2}$$

$$= 2 \cdot \frac{1}{3}t^{2} + 2$$
Using $x = \frac{1}{\sqrt{3}}t$, we have
 $z^{2} = 2x^{2} + 2$, and hence,
 $C : x^{2} - \frac{z^{2}}{2} = -1$

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 \rightarrow 3D viewer